Variationally Mimetic Operator Network with a Consistent Loss Function

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ABSTRACT

Deep learning-based operator networks have shown great promise in serving as efficient surrogates solvers for partial differential equations (PDEs). In a recent paper [1], a novel deep operator network was proposed that mimics the approximate variational or weak formulation of the PDE. This algorithm, termed as VarMiON, was empirically shown to be more robust and accurate compared to a standard DeepONet. The error analysis provided in [1] for VarMiONs was based on the error measured in the L_2 norm, and revealed the contribution from the error in the training data samples, the training error, and the discretisation/quadrature error at the input and output.

In this talk, we extend the training of the VarMiON by measuring the loss in a more appropriate Sobolev norm instead of the L_2 norm. This requires the solution and its derivatives to construct the training set, and a smoother network architecture to allow the computation of high-order derivatives of the operator network using back-propagation. We present the associated error analysis and investigate the numerical benefits with the extended VarMiON framework to solve elliptic PDEs. Further, we propose a series of numerical experiments, ranging from easy to challenging, which can serve as benchmark problems to assess the performance and robustness of operator networks.

REFERENCES

[1] D. Patel, D. Ray, M. R. A. Abdelmalik, T. J. R. Hughes, and A. A. Oberai. Variationally Mimetic Operator Networks, *ARXIV: 2209.12871*, (https://doi.org/10.48550/arxiv.2209.12871), 2022.