DIMENSIONALITY REDUCTION IN OPERATOR LEARNING

Jacob H. Seidman[1], Georgios Kissas[1], George J. Pappas[1], Paris Perdikaris[1]

[1]University of Pennsylvania

ABSTRACT

Part of the success of operator learning architectures such as the DeepONet [1] comes from their ability to act as universal approximators. However, universality alone is not sufficient for an operator learning architecture to be useful. For certain problems and architectures the model size needed for a prescribed accuracy can scale poorly, referred to as the curse of dimensionality. For example, this may occur when the output functions do not concentrate on low dimensional linear subspaces [2]. It was shown this particular difficulty can be avoided when the output functions instead concentrate on a low dimensional nonlinear manifold [3]. Therefore, the success of these architectures is dependent on how well they are able to perform dimensionality reduction for efficient finite dimensional encodings of input and output functions. This talk will explore examples of low dimensional structure in functional data for operator learning problems that can be leveraged for efficient representations. These examples will provide insight into what makes an operator learning benchmark challenging, as well as giving motivation for new architectures that can effectively deal with these challenges.

REFERENCES

[1] L. Lu, P. Jin, G. Pang, Z. Zhang, and G. E. Karniadakis, "Learning nonlinear operators via deeponet based on the universal approximation theorem of operators," Nature Machine Intelligence, vol. 3, no. 3, pp. 218–229, 2021.

[2] S. Lanthaler, S. Mishra, and G. E. Karniadakis, "Error estimates for deeponets: A deep learning framework in infinite dimensions," Transactions of Mathematics and Its Applications, vol. 6, no. 1, p. tnac001, 2022.

[3] J. H. Seidman, G. Kissas, P. Perdikaris, and G. J. Pappas, "NOMAD: Nonlinear manifold decoders for operator learning," in Advances in Neural Information Processing Systems, 2022.