USACM Workshop on Establishing Benchmarks for Data-Driven Modeling of Physical Systems
April 6-7, 2023, University of Southern California,, Los Angeles, California, USA

ON SOLVING/LEARNING NONLINEAR PDES WITH GPS

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ABSTRACT

We present a simple, rigorous, and unified framework for solving and learning arbitrary nonlinear PDEs with Gaussian Processes (GPs). The proposed approach: (1) provides a natural generalization of collocation kernel methods to nonlinear PDEs and Inverse Problems, (2) has guaranteed convergence for a very general class of PDEs, and (3) comes with a Bayesian interpretation compatible with a UQ pipeline. It inherits (1) the a priori error bounds of kernel interpolation methods and (2) the (near-linear) state-of-the-art computational complexity of linear solvers for dense kernel matrices. Its generalization to high-dimensional and parametric PDEs comes with error bounds exhibiting a tradeoff between dimensionality and regularity (the curse of dimensionality disappears when the problem is sufficiently regular). Its formulation can be interpreted and generalized as an extension of Gaussian Process Regression from the approximation of input/output functions to the completion of arbitrary computational graphs representing dependencies between multiple known and unknown functions and variables. Parts of this talk are joint work with Pau Batlle Franch, Yifan Chen, Bamdad Hosseini, Florian Schäfer, and Andrew Stuart.

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