Open Systems: Finite Metamaterials in Connection with Infinite Sinks

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ABSTRACT

In this talk, we will consider the problem of analyzing the dynamics of a finite system in connection with an infinite system (sink). The finite system may be of a usual nature, such as a beam capable of vibration, or of a more exotic nature, such as a finite sample of a metamaterial (conservative or non-conservative). The infinite system (bulk) is the environment in which the finite system is embedded.

When the finite system is coupled with the infinite system, it sets up some additional problems of dynamics. First, it becomes possible for the energy in the finite system to leak out into the environment. This leakage of energy from the finite system may be seen as the emergence of a non-conservative effect, even though there may not be any non-conservative sources in the finite system to begin with. This non-conservative effect is also captured in the dynamical eigenvalues of the finite system, which necessarily become complex (with the imaginary part connected to this fictitious damping effect) upon interaction with the environment. A related dynamic problem that emerges in this picture is that of scattering since the infinite environment is capable of sustaining wave phenomena which can interact with the finite system, resulting in scattered waves. All relevant information inherent in the scattering problem is captured in the scattering matrix, which exhibits peaks and valleys at precise frequencies. An obvious question is: what is the relation between the locations of the peaks and valleys of the scattering response (which occur in the real frequency domain) and the eigenvalues of the finite system (which are necessarily in the complex frequency domain, given the fictitious damping)?

We answer these and related questions through the formal machinery of open systems. Derived from quantum mechanics, the open system formalism provides a way to project the infinite-dimensional dynamics of the finite system + environment onto just the finite-dimensional dynamics of the finite system, thus making calculations possible. The connections between the scattering response, the eigenvalues of the finite system, and the overall Green's function become explicit in this formalism. Calculations can be carried out by resorting to some recent advances which go by the name of Atomistic Green's Functions. We adapt these advances, made originally in quantum mechanics, to classical vibrations and wave equations. The formalism proves an effective way of not only performing scattering calculations but also of understanding the inherent connections between vibration problems and wave problems..